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A novel consensus reaching method for MCGDM in social network with intuitionistic grey linguistic numbers

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Abstract

Purpose – The existing consensus reaching mechanisms ignore the influence of social triangle structure on the decision-makers' (DMs') weights, and the consensus reaching process (CRP) fails to fully reflect the DMs' subjectivity and can be time consuming and costly. To solve these issues, a novel CRP for multi-criteria group decision-making (MCGDM) problems with intuitionistic grey linguistic numbers (IGLNs) is proposed in this paper.

Design/methodology/approach – First, a weight calculation method is proposed by analysing the triangle structure of DMs' social network and scale of adjacent nodes. Then, a consensus degree index based on three-level polygon area is defined and applied to identify the inconsistent DMs. Finally, the feedback mechanism based on particle swarm optimisation (PSO) algorithm under grey linguistic environment is developed, where subjective trust relationships in social network is utilised to determine the adjustment coefficient.

Findings – The advantages of the proposed method are highlighted by two practical applications of the evaluation of tunnel construction method and the selection of a hotel for the centralised isolation. Comparision analysis and numerical simulation are performed to reveal the effectiveness and applicability of the method.

Practical implications – The proposed model can not only reflect the effect of triangle structure in social network on DMs' weights, but also reduce the time and cost of decision-making.

Originality/value – The main contribution of this paper is to propose a new MCGDM model based on intuitionistic grey linguistic numbers, which can handle the problem of inconsistency of information more effectively.

Keywords Multi-criteria group decision-making, Triangle structure, PSO algorithm, Intuitionstic grey linguistic numbers

Paper type Research paper

1. Introduce

Multi-criteria group decision-making (MCGDM) contains the evaluations of several decision-makers (DMs) on various feasible alternatives over different criteria. These evaluations from different DMs are aggregated into a group consensus to obtain an optimal alternative (Yu and Lai, 2011). Due to the complexity and uncertainty of research objects and the asymmetry of decision information, MCGDM problems always embody the dual characteristics of fuzziness and greyness. A variety of theories on uncertainty are emerging constantly including fuzzy theory and grey system theory. Fuzzy theory is widely used to handle the recognitive uncertainty (Zadeh, 1965). Fuzzy theory is closer to human mind and natural language system but it requires strong knowledge about the subject to make membership functions.

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Grey system theory is a new method to study uncertain problems with a few data and poor information. It works on uncertain systems with partially known and partially unknown information (Liu, 2017). Therefore, a combination of grey system theory and fuzzy theory can take the advantages of both to solve MCGDM problems. Different types of grey fuzzy numbers have been developed such as grey hesitant fuzzy numbers (Liu *et al.*, 2016) and intuitionistic grey numbers (Jiang *et al.*, 2020). However, with regard to qualitative decision information like company performance and management ability, linguistic term is closer to people's cognition than the crisp numerical value (Gou and Xu, 2021). In view of this, the interval grey linguistic variables (Wang *et al.*, 2019), three-parameter interval grey linguistic variables (Li and Yuan, 2017) and interval grey uncertain linguistic variables (Han *et al.*, 2016a) are put forward successively, which simultaneously consider the linguistic evaluations of DMs and the uncertainty degree of decision information. Although these linguistic variables can improve the accuracy of depicting complex grey fuzzy numbers, the incomplete degree of information cannot be reflected. To overcome this problem, intuitionistic grey linguistic numbers (IGLNs) is introduced into MCGDM (Li *et al.*, 2018), which have a strong ability to portray complex information.

Due to different knowledge backgrounds and professional levels of decision-makers, the inconsistency of information provided by DMs is frequently exists. To improve the quality of decision-making, the diverse opinions of all DMs needs to be coordinated to reach an acceptable level of consensus, and this multi-round dynamic interaction is known as the consensus reaching process (CRP) (Cheng *et al.*, 2018). A key issue to be addressed in CRP is the determination of DMs' weights. With the rapid development of communication and technology, it is increasingly common for DMs to participate in decision-making in the form of social network (such as Facebook, Weibo, and WeChat, etc.). The influence of DMs in social network provides a reliable source to determine the DMs' weights. Many centrality indices have been proposed to quantify the influence of DMs. For instance, the in-degree centrality is to determine the importance of DMs by social connections the DM has received from other DMs (Wu *et al.*, 2018). The out-degree centrality is to quantify social influence that the DM has sent to other DMs (Liang *et al.*, 2017; Chu *et al.*, 2016). The betweenness centrality is to measure DMs' influence by counting how many times a node appears on all the shortest paths between pairs of nodes (Lee *et al.*, 2021; Jung *et al.*, 2020). Although above centrality indices can measure the influence of DMs to a certain extent, the effect of structure of social network on DMs' weights is ignored. According to the structural hole theory, the individual's importance can be also reflected by his/her network topology structure (Burt, 1992; Cheng *et al.*, 2020). Triangle is an important structural feature in social network. The number of triangles between a DM and his/her adjacent DMs can measure the influence of a DM (Han *et al.*, 2016b). However, the scale of the member does not be reflected if only considering the structure between a DM and his/her adjacent DMs. Three degrees of influence rule points out that a DM can be affected not only by his/her adjacent DMs, but also by the adjacent DMs' adjacent DMs (Christakis and Fowler, 2009). To measure the influence of a DM more accurately, the scale of adjacent DMs also needs to be considered. In this paper, a novel weight determination method for DMs in MCGDM is proposed by analysing the triangular structure between DMs and the degree centrality of adjacent DMs.

Another typical issue in CRP is how to measure the closeness amongst DMs' evaluations in order to obtain the consensus level. There are two prevalent rules of consensus level measurement: one is based on the distance between individual decision matrices and group matrix, and the other is based on the distance between individual decision matrices (Herrera-Viedma *et al.*, 2014). The two rules calculate the consensus level from the perspective of distance, which only considers the proximity of two points on the distance. It is not ideal to consider only the point-to-point distance between criteria as a consensus level. In MCGDM, the optimal alternative is not only related to the distance between the corresponding attributes of the ideal alternative, but also related to each adjacent criterion (Jiang *et al.*, 2015). Especially for some subjective criteria, the interaction amongst criteria will be considered when DMs are evaluating. Therefore, a consensus degree index based on three-level polygon area is defined and applied to compute consensus level in this paper.

The last issue in CRP is how to design an appropriate feedback mechanism to modify DMs' evaluations with purpose of obtaining a high consensus level. Therefore, how to determine an optimal adjustment coefficient to make a balance between group consensus and individual opinion is a key issue in feedback mechanism. To do that, adjustment coefficient can be determined by mathematical models such as linear programming (Kamis *et al.*, 2018; Wu *et al.*, 2019), objective programming (Zhang and Pedrycz, 2018; Zhang and Xu, 2015), etc. However, the linear programming and objective programming functions may neglect the DMs' subjectivity, i.e. the traditional feedback mechanism does not consider whether the inconsistent DMs accept the recommended advices or not. To avoid this defect, some scholars have applied trust relationships in social network to improve CRP. Liu *et al.* (2017) developed a trust induced recommendation mechanism to generate personalised advice for the inconsistent experts to reach higher consensus degree. Wu *et al.* (2020) explored the effects of trust on consensus and some minimum cost consensus models are proposed based on implicit trust between individuals and the moderator. Although above trust-based feedback mechanism avoided the inconsistent DMs are forced to accept the recommended advices, the CRP are still time consuming and costly because the final evaluations are obtained after multi-round dynamic interaction. To improve consensus efficiency, the CRP algorithm should be further optimised. Particle swarm optimisation (PSO) is a simple but effective swarm intelligence optimisation, which has been proven to be an effective tool for streamlining decision-making. For instance, Liu and Yang (2022) proposed an extended VIKOR method based on the PSO algorithm for solving MCGDM problems with probabilistic linguistic information. Yan *et al.* (2022) employed PSO to improve the accuracy of information aggregation in the selection process. Garg and Rani (2021) proposed a new MCGDM method based on the non-linear programming methodology and PSO techniques with intuitionistic fuzzy sets. Therefore, it is feasible and effective to reduce the complexity of CRP and promote the performance of feedback mechanism by PSO algorithm. We design a novel feedback adjustment mechanism based on the PSO algorithm under intuitionistic grey linguistic environment, where trust relationships amongst DMs are used to derive the adjustment coefficient.

In this paper, we propose a novel consensus reaching process for MCGDM problems with IGLNs. Firstly, the influence of DMs in the social network is measured by the analysis of the triangle structure, and the DMs' weights are calculated by the Choquet integral operator. Then, a polygon area between adjacent points in the individual decision matrix and collective matrix is developed to measure the consensus level. For reflecting the DMs' subjectivity and simplifying the consensus reaching process, the particle swarm optimisation (PSO) algorithm that considers trust relationships is exploited to optimise the feedback mechanism. The proposed method is applied to two practical problems involving the evaluation of Mila mountain tunnel of Linzhi-Lhasa highway in Tibet and the selection of a hotel for the centralised isolation of entry personnel during the COVID-19 epidemic. By making comparative analyses with classical CRP method, the effectiveness and applicability of the proposed methods are highlighted.

The rest of this paper is set out as follows. Section 2 reviews the IGLNs definitions and operations. Section 3 gives the determination of DMs' weights by the social triangle structure analysis and Choquet integral operator. In Section 4, a MCGDM model based on novel consensus reaching process with IGLNs is developed. Section 5 illustrates the feasibility of the proposed method by two practical applications. Finally, the conclusions are presented in Section 6.

2. Preliminaries

In this section, we review some basic knowledge related to the IGLN, including the distance measure and score function.

2.1 Intuitionistic grey linguistic number

Definition 1. (Li *et al.*, 2018) Set the discourse domain X, and linguistic phase set $H = \{H_0, H_1, \ldots, H_{\tau-1}\}$. For $\forall x \in X$, there is a corresponding linguistic value $H_\alpha (H_\alpha \in H)$. If the membership degree $\mu(x)$ of x to H_α is the grey number in $[0, 1]$, its point grey scale is $v(x)$, then intuitionistic grey linguistic number (IGLN) on X is defined as follows:

$$A = \{x,\ H(x),\ \mu(x),\ v(x) | x \in X\}$$

where $\mu(x)$ is the membership degree, which can be understood as the close degree of x to H_α. $v(x)$ represents the extent to which the decision-makers' information is incomplete, namely, grey scale.

Remark 1. In general, the intuitionistic grey linguistic number A is abbreviated as $A = \{x, H(x), \mu(x), v(x) | x \in X\}$. If $a = \{H(x), \mu(x), v(x)\}$ is IGLN, any specific grey linguistic element a in A can be abbreviated as GLE. If the values of membership degree and grey scale in a is $\mu(x) = 1, v(x) = 0$, a does not have incomplete information degree, and retreats into the general linguistic number.

Example 1. Let H be a linguistic term set $H = \{H_0 = poor, H_1 = slightly\ poor, H_2 = fair, H_3 = slightly\ good, H_4 = good\}$, a teacher could evaluate a student by saying that he/she is "80% sure and 30% unsure that the attitude for study is good", the evaluations can be obtained as $A = \{H_4, 0.8, 0.3\}$.

Let $A = \{H_\alpha, \mu_\alpha, v_\alpha\}$ and $B = \{H_\beta, \mu_\beta, v_\beta\}$ be two IGLNs, then the operational rules are given below:

$$A + B = \left(H_{\alpha+\beta}, \mu_\alpha + \mu_\beta, max\left(v_\alpha, v_\beta\right)\right)$$

$$A \times B = \left(H_{\alpha \times \beta}, \mu_\alpha \mu_\beta, \max\left(v_\alpha, v_\beta\right)\right)$$

$$rA = \{H_{r \times \alpha}, r\mu_\alpha, v_\alpha\}$$

$$A^a = \left\{H_{\alpha^a}, \mu_\alpha^a,\ v_\alpha\right\}$$

Definition 2. (Wu, 2009) The distance between $A = \{H_\alpha, \mu_\alpha, v_\alpha\}$ and $B = \{H_\beta, \mu_\beta, v_\beta\}$ is defined as follows:

$$|A - B| = \left|\mu_\alpha I(H_\alpha) - \mu_\beta I\left(H_\beta\right)\right|_{max\left(v_\alpha, v_\beta\right)} \quad (1)$$

where I is the subscript operator of the $H_{\tau-1}$.

Example 2. In line with Example 1, the evaluation from the first teacher is $A = \{H_4, 0.8, 0.3\}$ and the evaluation for this student by another teacher is $B = \{H_3, 0.9, 0.2\}$and then the distance between A and B is calculated as follows:

$$|A - B| = |0.8 \times 4 - 0.9 \times 3|_{\max(0.3,0.2)} = 0.5_{0.3}$$

Definition 3. (Li *et al.*, 2018) Let $H = \{H_0, H_1, \ldots, H_{\tau-1}\}$ be a linguistic term set, $A = \{H_\alpha, \mu_\alpha, v_\alpha\}$ be an IGLN, then the score function of A can be defined as follows:

$$s(A) = (\mu_\alpha I(H_\alpha))_{v_\alpha} \tag{2}$$

Example 3. Following Example 1, the score for this student is as follows:

$$s(A) = (0.8 \times 4)_{0.3} = 3.2_{0.3}$$

3. Social network analysis based on triangle structure

Social network analysis (SNA) is regarded as one of the most popular methodologies for studying relationships amongst DMs. A DM with more connections (e.g. friendships or interactions) has more influential power than another DM with fewer connections. In social network, nodes and edges are used to represent DMs and their connections, respectively. There are three widely used modelling techniques (Sociometric, Graph theoretic and Algebraic) for social network analysis (Kundu and Pal, 2015), which show how to establish connections of DMs in a straightforward way. In this paper, we adopt Graph theoretical model to build the social relationships amongst DMs.

3.1 Degree centrality

The degree centrality is a key index to measure the influence of DMs, which is defined as follows.

Definition 4. (Luis *et al.*, 2014) Let $G = (E, L)$ be a graph which represent the topological structure of a social network, $E = \{e_1, e_2, \ldots, e_q\}$ be a set of nodes and $L = \{l_1, l_2, \ldots, l_p\}$ be the set of directed edges, between pairs of nodes where $|E| = q, |L| = p$ represent the graph G with q nodes and p edges. The degree centrality of the node e_k can be defined as follows:

$$D(e_k) = \sum_{k=1}^{q} z(e_k, e_h), \forall e_k, e_h \in E, k \neq h \tag{3}$$

where $z(e_k, e_h) = \begin{cases} 1, & \textit{there is a link between } e_k \textit{ and } e_h, \\ 0, & \textit{otherwise}. \end{cases}$

Degree centrality could measure the influence of DMs by the direct relationships between two DMs. However, the measure method based on degree centrality is insensitive because it cannot distinguish the importance of individuals whose degree centrality values are the same. The limitation of the degree centrality is illustrated by the following example.

The structure of the social relationship diagram of the poets in the prosperous Tang Dynasty is shown in Figure 1. Assume the undirected graph (see Figure 1) shows the connections amongst poets in social network. The social relationships amongst poets in Figure 1 are from China Biographical Database Project (CBDB), https://projects.iq.harvard.edu/cbdb. According to Eq. (1), we can obtain the poet e_{15} (Du Fu) and poet e_{19}

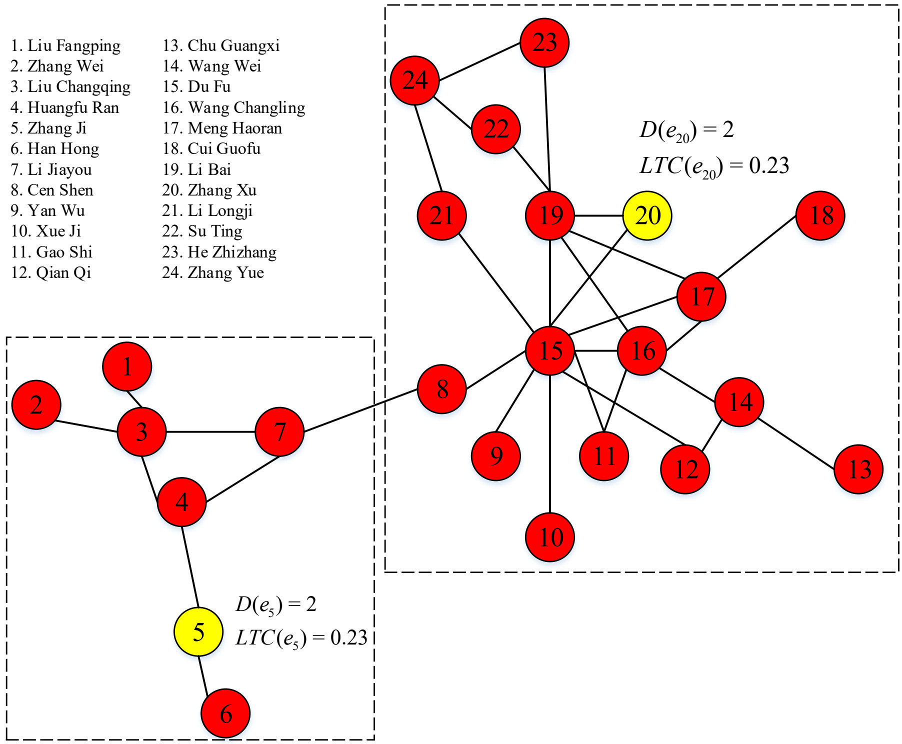

Figure 1.
The social relationship network of the poets in the prosperous Tang Dynasty

(Li Bai) with $D(e_{15}) = 10, D(e_{19}) = 6$ are the core poets of the social network respectively. However, for multiple poets with same and small degree values, the degree centrality cannot accurately distinguish the influence of the poets. For instance, the degrees of poets e_5 (Zhang Ji) and e_{20} (Zhang Xu) are both 2, i.e. $D(e_5) = D(e_{20}) = 2$, but the poet e_{20} interconnects with core poets e_{15} and e_{19} that forms a triangular structure. Therefore, the influence of poet e_{20} is obviously greater than that of poet e_5 at the edge of the network. To measure the influence of a DM more accurately, the degree information of adjacent DMs should also be considered.

3.2 Local triangle centrality

To avoid above limitation, the local triangle centrality (LTC) is introduced to measure the influence of DMs by combining the triangular structure and the degree information of adjacent DMs. The DM with a greater LTC is more important. The details are as follows.

Definition 4. Let $G = (E, L)$ be a graph which represent the topological structure of a social network, $E = \{e_1, e_2, ..., e_q\}$ be a set of nodes and $L = \{l_1, l_2, ..., l_p\}$ be the set of directed edges. Let $N(e_k)$ be a set of adjacent nodes, where $N(e_k) = \{e_v z(e_k, e_h) = 1, e_v \in E\}$, and the number of triangular structures formed between node e_k and node e_h, i.e. the number of common neighbour between node e_k and node e_h can be represented as $T(e_k, e_h)$, then its computing formula is as follows:

$$T(e_k, e_h) = |N(e_k) \cap N(e_h)| \tag{4}$$

Definition 5. (May, 1976) Suppose $S(x)$ is a Sigmod function that can map the value of $T(e_k, e_h)$ between 0 and 1, and the formula is as follows:

$$S(x) = (1 + \exp(-x))^{-1} \tag{5}$$

S function is a common S-shaped curve with the property of monotonically increasing, i.e. the more triangular structures between node e_k and its adjacent nodes, the greater the probability that node e_k is in the core area.

Definition 6. Combining the triangle structure with the characteristics of adjacent nodes, the local triangle centrality (*LTC*) of node e_k can be defined as follows:

$$LTC(e_k) = \frac{TC(e_k)}{\sqrt{\sum_{k=1}^{q} TC(e_k)^2}} \tag{6}$$

where

$$TC(e_k) = \sum_{v \in \{1,2,\ldots,q\}} S(T(e_k, e_v))D(e_v)^{\alpha} \tag{7}$$

The term $TC(e_k)$ denotes the relative triangle centrality of DM e_k and $\alpha \in [0, 1]$, $D(e_v)$ is the degree centrality of adjacent node e_v of node e_k. When $\alpha = 1$, i.e. the larger the degree of the adjacent node of node e_k, the greater the influence of node e_k.

Obviously, the local triangle centrality of DMs with the same degree centrality may be different in various surroundings. For instance, Table 1 gives the corresponding values of *D* and *LTC* for all poets in the social network. Taking the calculation of the *LTC* of the poet e_5 and poet e_{20} in Figure 1 as an example, the set of their adjacent poets are $N(e_5) = \{e_4, e_6\}$ and $N(e_{20}) = \{e_{15}, e_{19}\}$ respectively. Then the *LTC* of the poet e_5 and poet e_{20} can be obtained as follows:

$$TC(e_5) = S(T(e_5, e_4))D(e_4) + S(T(e_5, e_6))D(e_6) = 2,$$

$$TC(e_{20}) = S(T(e_{20}, e_{15}))D(e_{15}) + S(T(e_{20}, e_{19}))D(e_{19}) = 11.70,$$

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}
$D(e_k)$	1	1	4	3	2	1	3	2	1	1	2	2
$LTC(e_k)$	0.04	0.04	0.11	0.12	0.04	0.02	0.12	0.13	0.10	0.10	0.20	0.14
	e_{13}	e_{14}	e_{15}	e_{16}	e_{17}	e_{18}	e_{19}	e_{20}	e_{21}	e_{22}	e_{23}	e_{24}
$D(e_k)$	1	4	10	5	5	1	6	2	2	2	2	3
$LTC(e_k)$	0.04	0.17	0.42	0.47	0.42	0.05	0.41	0.23	0.13	0.09	0.09	0.06

Table 1. Comparison of *D* and *LTC* in a social network

$$LTC(e_5) = \frac{TC(e_5)}{\sqrt{\sum_{k=1}^{24} TC(e_1)^2 + TC(e_2)^2 + \ldots + TC(e_{24})^2}} = 0.04,$$

$$LTC(e_{20}) = \frac{TC(e_{20})}{\sqrt{\sum_{k=1}^{24} TC(e_1)^2 + TC(e_2)^2 + \ldots + TC(e_{24})^2}} = 0.23.$$

From Table 1, for the poets with the same degree centrality such as e_5, e_{20}, e_4 and e_{24}, $D(e_k)$ does not distinguish the influence of poets, and $LTC(e_k)$ can accurately measure their influence.

3.3 Choquet integral operator

As mentioned above, the influence of each DM could be determined according to his/her LTC. A DM with a higher LTC indicates a more significant location and a larger weight on the social network. However, in real social network, due to the non-strictly independent and interactions amongst DMs, DMs' weights do not satisfy additivity, which may lead to the phenomenon of the failure of the general weighting operators. In order to derive more accurate weights of DMs, the interactions amongst DMs should be considered. Murofushi and Sugeno proposed the Choquet integral operator on fuzzy measures, which has the ability to capture the interactions amongst objects.

Definition 7. (Murofushi and Sugeno, 1989) Let $E = \{e_1, e_2, \ldots e_q\}$ be a set of DMs. Suppose that $\Phi(E)$ is the power set of E and that $\varphi : \Phi(E) \to [0, 1]$ is called as fuzzy measure set function, which satisfies the following axioms:

(1) $\varphi(\varnothing) = 0, \varphi(E) = 1$;

(2) $\forall A, B \in \Phi(E)$, if $A \subseteq B$, then $\varphi(A) \leq \varphi(B)$;

(3) $\varphi(A \cup B) = \varphi(A) + \varphi(B) + \theta\varphi(A)\varphi(B)$, where $\theta \in [-1, \infty)$.
Then φ is said to be the θ-fuzzy measure on E.

In the MCGDM problem, the interacting amongst DMs can be expressed more accurately by using θ-fuzzy measure. If E is a set of DMs of a MCDM problem, $A, B \in \Phi(E)$, $\varphi(A)$ and $\varphi(B)$ can be considered as the weight of DMs set A and B.

If $\theta = 0$ and $\varphi(A \cup B) = \varphi(A) + \varphi(B)$, then DMs set A and B are independent of each other; If $-1 \leq \theta \leq 0$ and $\varphi(A \cup B) < \varphi(A) + \varphi(B)$ then redundant association between DMs sets exists; If $\theta > 0$ and $\varphi(A \cup B) > \varphi(A) + \varphi(B)$, then complementary association between DMs sets exists.

Definition 8. Suppose that f is a positive real-valued function on $E = \{e_1, e_2, \ldots, e_q\}$, and that φ is a fuzzy measure on E. According to fuzzy measure φ, the Choquet integral function is defined as follows:

$$\int f d\varphi = \sum_{k=1}^{q} f\left(e_{(k)}\right)\left[\varphi\left(F_{(k)}\right) - \varphi\left(F_{(k+1)}\right)\right] \tag{8}$$

where (k) denotes the subscript sorted by $f(e_{(1)}) \leq f(e_{(2)}) \leq \ldots \leq f(e_{(q)})$, $F_{(q+1)} = 0$ and $F_{(k)} = \{e_{(k)}, e_{(k+1)}, \ldots, e_{(q)}\}$.

4. MCGDM model based on novel consensus reaching process

The flowchart of the proposed method is shown in Figure 2, and specific steps of the method are summarised as follows: To summarise, the following seven steps are listed:

Step 1. Establish DMs' individual decision matrices, and evaluation values are given using IGLNs.

Step 2. Calculate the influence of each DM by Eq. (7), then compute DMs' weights by Eq. (11).

Step 3. Aggregate the individual decision matrices by Eq. (12).

Step 4. Compute consensus degree of each DM by Eqs. (14)-(16). If $CD^k \geq \gamma$, then go to step 7; otherwise, go on.

Step 5. Obtain the trust matrix based on social network, then the adjustment coefficients are determined based on trust relationship in social network by Eq. (20).

Step 6. Modify the evaluations of the inconsistent DMs by Eqs.(21)-(22), then go to Step 3.

Step 7. Calculate the dominance degrees for the alternatives by Eq. (25), rank the alternatives and finally select the optimal alternative.

4.1 Problem description

Let $A_i = \{A_1, A_2, ..., A_n\}$ be a discrete set of alternatives, $C_j = \{c_1, c_2, ..., c_m\}$ be a set of predefined criteria and $\omega_j = (\omega_1, \omega_2, ..., \omega_m)^T$ be the criteria weight vectors, where $\sum_{j=1}^{m} \omega_j = 1$. Let $E = \{e_1, e_2, ..., e_q\}$ be a set of DMs and $\lambda_k = (\lambda_1, \lambda_2, ..., \lambda_q)$ be the weight vectors of DMs. Let $X^k = (x_{ij}^k)_{n \times m}$, $x_{ij}^k = \{H_\alpha, \mu_{ij}^k, v_{ij}^k\}$ be a decision matrix, where LGLN x_{ij}^k denotes the evaluation of alternative A_i for attribute c_j, given by DM e_k.

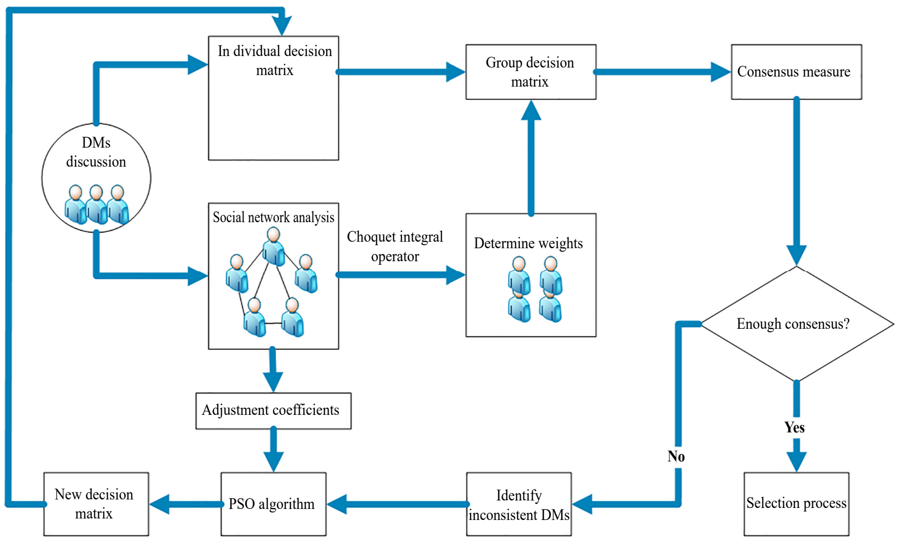

Figure 2.
The flowchart of the proposed method

4.2 Determination of DMs' weights

Inspired by the thought of Murofushi and Sugeno (1989), a Choquet integral operator is introduced to derive the DMs' weights. Taking the correlation amongst DMs into consideration, the fuzzy measure of each DM needs to be determined. In this paper, the corresponding LTC value of each DM is set as the fuzzy measure. Then, the parameter θ in the θ-fuzzy measure can be determined by Eqs. (9)-(10).

$$\varphi(E) = \begin{cases} \frac{1}{\theta}\left[\prod_{k=1}^{q}(1+\theta\varphi(e_k)) - 1\right], \theta \neq 0 \\ \prod_{k=1}^{q}\varphi(e_k), \qquad \theta = 0 \end{cases} \tag{9}$$

where $e_k \cap e_h = \varnothing$, $k \neq h$. For a subset with only one DM e_k, $\varphi(e_k)$ is called a fuzzy measure, then $\varphi_k = \varphi(e_k)$. From the marginal condition $\varphi(E) = 1$, it can be obtained:

$$\theta = \prod_{k=1}^{q}(1+\theta\varphi(e_k)) - 1 \tag{10}$$

Let $\lambda_k = (\lambda_1, \lambda_2, \ldots, \lambda_q)$ be the weight vectors of DMs, then the weight of DM e_k based on the social network triangular structure and Choquet integral operator is obtained as follows:

$$\lambda_k = \varphi\left(F_{(k)}\right) - \varphi\left(F_{(k+1)}\right), k = 1, 2, \ldots, q \tag{11}$$

4.3 Aggregation the decision matrix of each DM

After obtaining the DMs' weights, group decision matrix can be calculated by aggregating individual decision matrices. In this model, group decision matrix $X^c = (x_{ij}^c)_{n\times m}$ is obtained by Choquet integral operator to aggregate the individual decision matrices $X^k = (x_{ij}^k)_{n\times m}$. The specific calculation method is as follows:

$$x_{ij}^c = \left\{ H_{\sum_{k=1}^{q}\lambda_k\alpha}, \sum_{k=1}^{q}\lambda_k\mu_{ij}^k, \max_{1\leq k\leq q} v_{ij}^k \right\} \tag{12}$$

where $\sum_{k=1}^{q}\lambda_k = 1$.

4.4 A novel consensus measure based on area

Once the group decision matrix is calculated by Eq. (12), the next step is to measure the consensus degree of each DM to the group. A new consensus measure method based on area, consensus degree (CD), is used to measure the consensus level.

Let $X^k = (x_{ij}^k)_{n\times m}$ be the individual decision matrices, and $X^c = (x_{ij}^c)_{n\times m}$ be group decision matrix. The area function between two grey linguistic set functions is introduced to determine the consensus degree of the two elements of alternatives A_i for attribute c_j.

$$\rho\left(x_{ij}^k, x_{ij}^c\right) = \int_j^{j+1}\left|x_{ij}^k - x_{ij}^c\right| dt = \frac{\left|x_{i(j+1)}^k - x_{i(j+1)}^c\right| + \left|x_{ij}^k - x_{ij}^c\right|}{2} \tag{13}$$

Proof. (1) When the line between point (j, x_{ij}^c) and point $(j+1, x_{i(j+1)}^c)$ does not intersect the line between point (j, x_{ij}^k) and point $(j+1, x_{i(j+1)}^k)$, the line amongst the four points forms a trapezoid. According to the trapezoid area formula:

$$\rho\left(x_{ij}^k, x_{ij}^c\right) = \int_j^{j+1} \left|x_{ij}^k - x_{ij}^c\right| dt = \frac{\left|x_{i(j+1)}^k - x_{i(j+1)}^c\right| + \left|x_{ij}^k - x_{ij}^c\right|}{2}$$

(2) A triangle is formed when the line between point (j, x_{ij}^c) and point $(j+1, x_{i(j+1)}^c)$ intersects the line between point (j, x_{ij}^k) and point $(j+1, x_{i(j+1)}^k)$ at an endpoint. According to the triangle area formula:

$$\rho\left(x_{ij}^k, x_{ij}^c\right) = \int_j^{j+1} \left|x_{ij}^k - x_{ij}^c\right| dt = \frac{\left|x_{i(j+1)}^k - x_{i(j+1)}^c\right|}{2},$$

or

$$\rho\left(x_{ij}^k, x_{ij}^c\right) = \frac{\left|x_{ij}^k - x_{ij}^c\right|}{2}.$$

is a special case, so the theorem holds.

The consensus degree amongst DMs can be obtained by calculating three different levels of consensus degree:

Level 1. (Consensus degree for element). The consensus degree between DM e_k and group at the element level (A_i, c_j) is as follows:

$$CD_{ij}^k = 1 - \rho\left(x_{ij}^k, x_{ij}^c\right) = 1 - \frac{\left|x_{i(j+1)}^k - x_{i(j+1)}^c\right| + \left|x_{ij}^k - x_{ij}^c\right|}{2} \tag{14}$$

Level 2. (Consensus degree for alternative). The consensus degree between DM e_k and group at the alternative level (A_i) is as follows:

$$CD_i^k = 1 - \frac{1}{m-1} \sum_{j=1}^{m} \frac{\left|x_{i(j+1)}^k - x_{i(j+1)}^c\right| + \left|x_{ij}^k - x_{ij}^c\right|}{2} \tag{15}$$

Level 3. (Consensus degree for individual). The consensus degree between DM e_k and group at decision matrix level is as follows:

$$CD^k = \frac{1}{n} \sum_{i=1}^{n} CD_i^k = 1 - \frac{1}{n(m-1)} \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\left|x_{i(j+1)}^k - x_{i(j+1)}^c\right| + \left|x_{ij}^k - x_{ij}^c\right|}{2} \tag{16}$$

The larger the value of CD^k, the higher the consensus degree between e_k and the group. When $CD^k = 1$, it means that DM e_k is full consensus with the group. However, it is almost impossible in a real-life GDM problem. Therefore, it is necessary to determine the tolerance of deviation between individual evaluations and group evaluations, i.e. to determine the consensus threshold γ. The accurate selection of consensus threshold directly affects the quality of decision-making, but there are no unified methods to set the threshold. According to the specific requirements of the amendment of constitution, the passing of constitutional amendments must be approved by a two-thirds majority of all NPC deputies (Hou and Tong, 2021). Obviously, 2/3 is the lower limit of a high consensus degree in democratic practice. Therefore, the threshold is set as $\gamma = 0.6700$ in this paper. When $CD^k < \gamma$, a feedback mechanism could be activated.

4.5 Feedback mechanism based on PSO algorithm

During the feedback process of CRP, the DMs are allowed to modify their evaluations for reaching higher consensus level. Particle swarm optimisation (PSO) was first proposed in 1995 to optimise non-linear functions (Eberhart and Kennedy, 1995). PSO has since been proven to be an effective tool for streamlining decision-making. In the PSO, each particle has both position and velocity vectors, with the position vector representing the current solution for the corresponding particle and the velocity vector being utilised to provide direction and to adjust the particle to the optimal solution position. In this GDM with PSO, the DMs are regarded as particles and the decision matrices are regarded as the particle positions. Each PSO particle moves toward the optimal position, which represents the final evaluations that have reached the predefined consensus threshold. Then the modification process is shown.

In the traditional PSO algorithm, it is presumed that the position and velocity of a particle are denoted as $p(t)$ and $v(t)$ at tth iteration, respectively. Then the updates of the particle's velocity and position are realised in the form of Eqs.(17) and (18).

$$v(t+1) = v(t) + c_1 r_1 \left(x_{ij}^{k\,best}(t) - x_{ij}^{k}(t) \right) - c_2 r_2 \left(x_{ij}^{c\,best}(t) - x_{ij}^{k}(t) \right) \tag{17}$$

$$p(t+1) = p(t) + v(t+1) \tag{18}$$

where t denotes the round of iteration, $p(t+1)$ is the new position of particle, c_1 and c_2 are constant and defined as the learning factors. $r_1, r_2 \in [0,1]$ are random numbers that represent the diversity of the algorithm, $x_{ij}^{k\,best}$ and $x_{ij}^{c\,best}$ are the personal best position (solution) and the global best solution, respectively.

To revise the initial evaluations and obtain the new evaluations, each DM's tendency to modify their evaluations needs to be quantified. Therefore, the next section shows how the trust relationship-based social network can be used to determine the adjustment coefficient.

The trust relationship-based social network could not only be used to derive the DMs' weights, but also effect the CRP (Samadi *et al.*, 2016). The more DM e_k trusts DM e_h, the more likely DM e_k is to modify his/her evaluations to be line with DM e_h (Li *et al.*, 2013). Let e_k and e_h be any two DMs, and u_{kh} represent the trust degree that DM e_k allocates to DM e_h . Due to the complexity of social relationships, some DMs may express their trust degrees as "low" or "high" associated with different membership degree and uncertainty degree, rather than in the form of "trusting" or "distrusting". Then IGLNs can be an effective expression in describing the trust degrees the trust degree u_{kh} between e_k and e_h. The trust relationship can be denoted as the trust matrix U, which is given below:

$$U = (u_{kh})_{q \times q} = \begin{bmatrix} u_{11} & u_{12} & \dots & u_{1q} \\ u_{21} & u_{22} & \dots & u_{2q} \\ \dots & \dots & \dots & \dots \\ u_{q1} & u_{q2} & \dots & u_{qq} \end{bmatrix} \tag{19}$$

As the trust degree u_{kh} is represented by IGLN, to convert it into a real number, the score function is employed to express the corresponding trust score s_{kh}. Then, Eq. (20) is used to calculate the adjustment coefficient:

$$\delta_{kh} = s_{kh} \Big/ \sum_{k=1}^{q} s_{kh}, \forall e_k, e_h \in E, k \neq h \tag{20}$$

where δ_{kh} is the adjustment coefficient for e_k to e_h. It represents the adjustment tendency of DM e_k after knowing the opinion of DM e_h . $s_{kh} = s(u_{kh})$, $s(\cdot)$ is the score function of IGLNs presented in Eq. (2). Through the improved update function, we can produce a new evaluation of DM e_k. The update functions for the velocity and position are presented as follows:

$$v^k(t+1) = \sum_{k=1}^{q} \delta_{kh} X^k(t) \ (k \neq h) \tag{21}$$

$$\overline{X}^k(t+1) = \delta_{kh} X^k(t) + v^k(t+1) \tag{22}$$

where $v^k(t+1)$ and $\overline{X}^k(t+1)$ represent the velocity and the position of DM e_k in the $t+1$ th round, respectively.

Algorithm 1: The PSO algorithm under the intuitionstic grey linguistic environment

Input: the decision matrix of DM, adjustment coefficient δ_{kh} for e_k to e_h, and consensus threshold γ

Output: The final evaluations $X^1(t), X^2(t), ..., X^q(t)$ of DMs $e_1, e_2, ..., e_q$, which achieve the requirement of consensus level

Initialization: Let the iteration $t = 2$ and initial position for particle $X^1(t)$ is a cell array obtained from the DM's decision matrix, $k = 1, 2, ..., q$;
Calculate each particle's initial consensus levels $CD^k(1)$ by Eq. (16), which is also its best fitness CD^{kbest};
while $\min(CD^{kbest}) < \gamma$ **do**
  for each particle $X^k(t)$ **do**
    if $CD^k(t) < \gamma$ **then**
      Update $v^k(t)$ and $X^k(t)$ by Eqs. (21)-(22);
    else
      $v^k(t) = v^k(t-1)$ and $X^k(t) = X^k(t-1)$;
  for each particle $X^k(t)$ **do**
    Calculate each particle's new consensus levels $CD^k(t)$ of his/her evaluations by Eq.(16);
    if $CD^k(t) > CD^{kbest}$ **then**
      $CD^{kbest} = CD^k(t)$;
    else
      $v^k(t) = v^k(t-1)$ and $CD^k(t) = CD^{kbest}$;
  $t = t + 1$.

If $CD_k \geq \gamma$, the overall consensus level is considered acceptable. The next step is to move to selection process.

4.6 Selection process

Let $\overline{X}^c = (\overline{x}_{ij}^c)_{n\times m}$ be the new group decision matrix and $\omega_j = (\omega_1, \omega_2, \ldots, \omega_m)^T$ be the criteria weight vector. Based on Eq. (2), the positive ideal scheme z^+ and the negative ideal scheme z^- are determined as follows:

$$\begin{cases} z^+ = \{z_1^+, z_2^+, \ldots, z_m^+\} = \left\{\max_i \overline{x}_{i1}^c, \max_i \overline{x}_{i2}^c, \ldots, \max_i \overline{x}_{im}^c\right\} \\ z^- = \{z_1^-, z_2^-, \ldots, z_m^-\} = \left\{\min_i \overline{x}_{i1}^c, \min_i \overline{x}_{i2}^c, \ldots, \min_i \overline{x}_{im}^c\right\} \end{cases} \tag{23}$$

The relative weight ω_{jr} of attribute c_j to the reference attribute c_r is calculated according to the following Eq. (24).

$$\omega_{jr} = \omega_j / \omega_r \tag{24}$$

where $\omega_r = \max\{\omega_j | j = 1, 2\ldots, m\}$.

The positive (negative) dominance degree of each alternative A_i is expressed as follows:

$$\begin{cases} \vartheta_i^+ = \sum_{j=1}^{m} \psi_i^{j(+)} \\ \vartheta_i^- = \sum_{j=1}^{m} \psi_i^{j(-)} \end{cases} \tag{25}$$

where

$$\begin{cases} \psi_i^{j(+)} = \sqrt{\omega_{jr} \left| \overline{x}_{ij}^c - z_j^- \right| \Big/ \sum_{j=1}^{m} \omega_{jr}} \\ \psi_i^{j(-)} = -\frac{1}{\xi} \sqrt{\left(\sum_{j=1}^{m} \omega_{jr}\right) \left| \overline{x}_{ij}^c - z_j^+ \right| \Big/ \omega_{jr}} \end{cases} \tag{26}$$

The term $\psi_i^{j(+)}$ ($\psi_i^{j(-)}$) denotes the dominance degree of each scheme A_i over positive (negative) ideal scheme z^+ (z^-) with respect to attribute c_j. The parameter ξ denotes loss aversion index which can be tuned according to the problem at hand.

The higher ϑ_i^+ and the lower ϑ_i^- are, the better the alternative A_i is.

5. Application examples

To verify the effectiveness of the proposed method, two application examples of the evaluations of tunnel construction method and a hotel for the centralised isolation are given in this section.

5.1 Evaluation of tunnel construction method

5.1.1 Overview of the project. Mila Mountain tunnel, one of the key highway renovation projects in the section of Nyingchi to Lhasa belonging to the national highway numbered as 318 route in Tibet located at the junction of Gongbo'gyamda County and Maizhokunggar County where the average elevation is 4763.5 m, and the maximum reaches 5,020 m, characterised by thin air, low pressure, poor nature, and geological conditions. It is an extra-long highway tunnel at the highest elevation in the world designed with double holes in separation for 30~39 m in distance, 5,727 m in length for the left hole, and 5,720 m for the right, about 375 m at the maximum depth, 5.0 m for the height, and 10.25 m for the width. The surrounding rocks of the tunnel are dominated IV and V grade tuffs, accounting for about 85%. The section ZK4476 + 964−ZK4477 + 331 (see Figure 3) on the left line of Milla mountain tunnel is selected as the study case, and the tunnel construction methods are optimised and analysed. The surrounding rock of this section is V grade tuff, with developed fissure water and abundant groundwater. According to the characteristics of the surrounding rock of this section of the tunnel, after consulting with tunnel experts and tunnel construction units, it is preliminarily determined that there are four tunnel construction methods: ring cut method (A_1), centre diaphragm (CD) method (A_2), cross diaphragm (CRD) method (A_3) and double sidewall pilot pit excavation method (A_4). To make a reasonable evaluation of the tunnel construction methods, five evaluation criteria are selected here: construction environment (c_1), economy (c_2), construction technology difficulty (c_3), construction progress (c_4), and construction mechanisation level (c_5). To ensure construction safety and progress, a decision-making and command centre composed of five departments were set up. The five departments were engineering management department, quality and safety department, equipment and materials department, financial management department and scientific research department. To make the decision, five experts are invited to evaluate the four methods, who are from above five departments, the data are derived from Xie and Ding (2019).

Step 1. Each DM provided an initial evaluation matrix for the four alternatives with respect to the five criteria. The linguistic term set is $H = \{H_0 = poor, H_1 = slightly\ poor, H_2 = fair, H_3 = slightly\ good, H_4 = good\}$. The five decision matrices with intuitionistic grey linguistic numbers are shown below:

$$X^1 = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 & c_5 \\ A_1 & \{H_0, 0.2, 0.3\} & \{H_2, 0.6, 0.2\} & \{H_2, 0.6, 0.1\} & \{H_2, 0.6, 0.1\} & \{H_2, 0.6, 0.3\} \\ A_2 & \{H_1, 0.4, 0.2\} & \{H_1, 0.4, 0.4\} & \{H_1, 0.3, 0.1\} & \{H_1, 0.4, 0.3\} & \{H_1, 0.4, 0.5\} \\ A_3 & \{H_2, 0.5, 0.3\} & \{H_0, 0.1, 0.1\} & \{H_0, 0.2, 0.2\} & \{H_0, 0.1, 0.1\} & \{H_1, 0.4, 0.3\} \\ A_4 & \{H_3, 0.7, 0.1\} & \{H_0, 0.1, 0.3\} & \{H_0, 0.1, 0.1\} & \{H_0, 0.1, 0.3\} & \{H_0, 0.1, 0.2\} \end{bmatrix}$$

$$X^2 = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 & c_5 \\ A_1 & \{H_0, 0.2, 0.1\} & \{H_3, 0.7, 0.1\} & \{H_2, 0.6, 0.2\} & \{H_2, 0.6, 0.2\} & \{H_3, 0.7, 0.2\} \\ A_2 & \{H_1, 0.3, 0.2\} & \{H_2, 0.5, 0.2\} & \{H_1, 0.3, 0.1\} & \{H_2, 0.5, 0.2\} & \{H_1, 0.4, 0.4\} \\ A_3 & \{H_2, 0.5, 0.1\} & \{H_1, 0.3, 0.3\} & \{H_0, 0.2, 0.5\} & \{H_1, 0.3, 0.1\} & \{H_1, 0.3, 0.1\} \\ A_4 & \{H_3, 0.8, 0.2\} & \{H_1, 0.3, 0.7\} & \{H_0, 0.2, 0.1\} & \{H_0, 0.2, 0.2\} & \{H_0, 0.2, 0.3\} \end{bmatrix}$$

Figure 3.
The left line of Milla mountain tunnel

$$X^3 = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 & c_5 \\ A_1 & \{H_0, 0.2, 0.3\} & \{H_3, 0.7, 0.1\} & \{H_3, 0.7, 0.1\} & \{H_2, 0.5, 0.3\} & \{H_2, 0.6, 0.1\} \\ A_2 & \{H_1, 0.3, 0.1\} & \{H_1, 0.4, 0.2\} & \{H_1, 0.4, 0.3\} & \{H_1, 0.4, 0.4\} & \{H_1, 0.4, 0.3\} \\ A_3 & \{H_2, 0.5, 0.1\} & \{H_0, 0.2, 0.2\} & \{H_1, 0.3, 0.3\} & \{H_0, 0.2, 0.2\} & \{H_1, 0.4, 0.3\} \\ A_4 & \{H_3, 0.7, 0.5\} & \{H_0, 0.2, 0.1\} & \{H_0, 0.2, 0.2\} & \{H_0, 0.2, 0.2\} & \{H_0, 0.1, 0.1\} \end{bmatrix}$$

$$X^4 = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 & c_5 \\ A_1 & \{H_0, 0.2, 0.2\} & \{H_2, 0.6, 0.1\} & \{H_2, 0.6, 0.3\} & \{H_2, 0.5, 0.2\} & \{H_3, 0.7, 0.1\} \\ A_2 & \{H_0, 0.2, 0.1\} & \{H_1, 0.3, 0.4\} & \{H_2, 0.5, 0.3\} & \{H_2, 0.5, 0.2\} & \{H_2, 0.6, 0.4\} \\ A_3 & \{H_2, 0.6, 0.2\} & \{H_0, 0.1, 0.4\} & \{H_1, 0.4, 0.2\} & \{H_1, 0.3, 0.3\} & \{H_1, 0.4, 0.6\} \\ A_4 & \{H_2, 0.6, 0.3\} & \{H_0, 0.2, 0.2\} & \{H_1, 0.3, 0.2\} & \{H_1, 0.3, 0.1\} & \{H_1, 0.4, 0.4\} \end{bmatrix}$$

$$X^5 = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 & c_5 \\ A_1 & \{H_0, 0.1, 0.2\} & \{H_3, 0.8, 0.2\} & \{H_2, 0.6, 0.2\} & \{H_3, 0.7, 0.3\} & \{H_3, 0.8, 0.3\} \\ A_2 & \{H_1, 0.3, 0.1\} & \{H_2, 0.5, 0.5\} & \{H_2, 0.5, 0.4\} & \{H_2, 0.5, 0.1\} & \{H_2, 0.5, 0.4\} \\ A_3 & \{H_1, 0.4, 0.3\} & \{H_1, 0.3, 0.2\} & \{H_1, 0.4, 0.2\} & \{H_1, 0.4, 0.2\} & \{H_0, 0.2, 0.3\} \\ A_4 & \{H_2, 0.6, 0.7\} & \{H_1, 0.4, 0.3\} & \{H_1, 0.4, 0.1\} & \{H_1, 0.4, 0.1\} & \{H_0, 0.2, 0.3\} \end{bmatrix}$$

Step 2. The social influence of each DM is obtained by social relationship amongst DMs (see Figure 4), then the weight of each DM can be derived.

The social influence of each DM is calculated based on Eq. (6), and the results are: $LTC(e_1) = 0.52$, $LTC(e_2) = 0.44$, $LTC(e_3) = 0.56$, $LTC(e_4) = 0.44$, and $LTC(e_5) = 0.17$. Then, the social influence value of each DM is set as the fuzzy measure of each DM.

According to Eq. (9), we can get:

$$\frac{1}{\theta_1}[(1 + 0.52\theta_1)(1 + 0.44\theta_1)(1 + 0.56\theta_1)(1 + 0.44\theta_1)(1 + 0.17\theta_1) - 1] = 1.$$

Then, solving the above equation by Matlab, $\theta_1 = -0.926$ can be obtained, then the fuzzy measure (weight) of each DM subset of attribute set $E = \{e_1, e_2, e_3, e_4, e_5\}$ is calculated (Table 2).

By using $\lambda_k = \mu(F_k) - \mu(F_{k+1})$ in Eq. (11), the weights of five DMs can be derived as follows:

Lack of the weight of DM e_4

$$\lambda_1 = 0.07, \lambda_2 = 0.11, \lambda_3 = 0.28, \lambda_4 = 0.37, \lambda_5 = 0.17.$$

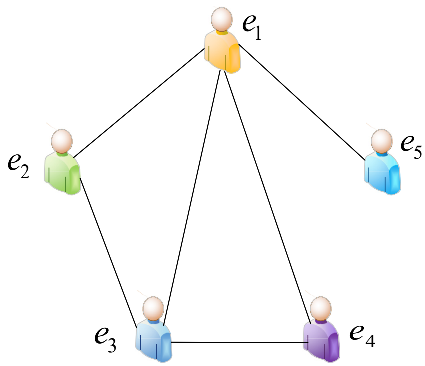

Figure 4.
The undirected social network

Step 3. The group decision matrix is calculated by Eq. (12).

$$X^c = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 & c_5 \\ A_1 & \{H_0,0.18,0.3\} & \{H_{2.6},0.67,0.2\} & \{H_{2.3},0.63,0.3\} & \{H_{2.2},0.55,0.3\} & \{H_{2.7},0.68,0.3\} \\ A_2 & \{H_{0.6},0.27,0.2\} & \{H_{1.3},0.39,0.5\} & \{H_{1.5},0.44,0.4\} & \{H_{1.7},0.47,0.4\} & \{H_{1.5},0.49,0.5\} \\ A_3 & \{H_{1.8},0.52,0.9\} & \{H_{0.3},0.18,0.4\} & \{H_{0.8},0.34,0.5\} & \{H_{0.7},0.28,0.3\} & \{H_{0.8},0.36,0.6\} \\ A_4 & \{H_{2.5},0.66,0.8\} & \{H_{0.3},0.24,0.7\} & \{H_{0.5},0.26,0.2\} & \{H_{0.5},0.26,0.3\} & \{H_{0.4},0.24,0.4\} \end{bmatrix}$$

Step 4. The consensus degrees of five DMs are computed based on Eqs. (14)-(16), and the results are $CD^1 = 0.7867$, $CD^2 = 0.7668$, $CD^3 = 0.7813$, $CD^4 = 0.7883$, and $CD^5 = 0.6579$.

Due to $CD^5 < 0.6700$, therefore, DM e_5 needs to modify her/his evaluations to reach the threshold.

Step 5. The DMs express their trust in each other and the trust matrix is obtained as follows:

$$U = \begin{bmatrix} \{H_2,1,0\} & \{H_1,1,0\} & \{H_4,1,0\} & \{H_3,1,0\} & \{H_1,1,0\} \\ \{H_1,1,0\} & \{H_2,1,0\} & \{H_4,1,0\} & - & - \\ \{H_0,1,0\} & \{H_1,1,0\} & \{H_2,1,0\} & \{H_1,1,0\} & - \\ \{H_1,1,0\} & - & \{H_3,1,0\} & \{H_2,1,0\} & - \\ \{H_1,1,0\} & - & - & - & \{H_2,1,0\} \end{bmatrix}$$

Based on the score function, the intuitionistic grey linguistic numbers are converted into crisp values as shown below:

$$U = \begin{bmatrix} 2 & 1 & 4 & 3 & 1 \\ 1 & 2 & 4 & - & - \\ 0 & 1 & 2 & 1 & - \\ 1 & - & 3 & 2 & - \\ 1 & - & - & - & 2 \end{bmatrix}$$

The adjustment coefficients for e_5 are determined using Eq. (20) as follows: $\delta_{51} = 0.33$, $\delta_{52} = 0$, $\delta_{53} = 0$, $\delta_{54} = 0$, $\delta_{55} = 0.67$.

Step 6. According to Eqs. (20)-(21), the revised functions are applied to modify the evaluations of e_5.

$$v^5(2) = \delta_{51}X^1(1) + \delta_{52}X^2(1) + \delta_{53}X^3(1) + \delta_{54}X^4(1)$$

$$X^5(2) = \delta_{55}X^5(1) + v^5(2)$$

DM subset	Weight	DM subset	Weight	DM subset	Weight	DM subset	Weight
$\{e_1,e_2\}$	0.75	$\{e_3,e_4\}$	0.73	$\{e_1,e_3,e_5\}$	0.88	$\{e_1,e_2,e_3,e_5\}$	0.96
$\{e_1,e_3\}$	0.81	$\{e_3,e_5\}$	0.66	$\{e_1,e_4,e_5\}$	0.77	$\{e_1,e_2,e_4,e_5\}$	0.90
$\{e_1,e_4\}$	0.75	$\{e_4,e_5\}$	0.54	$\{e_2,e_3,e_4\}$	0.88	$\{e_1,e_3,e_4,e_5\}$	0.94
$\{e_1,e_5\}$	0.61	$\{e_1,e_2,e_3\}$	0.92	$\{e_2,e_3,e_5\}$	0.84	$\{e_2,e_3,e_4,e_5\}$	0.93
$\{e_2,e_3\}$	0.77	$\{e_1,e_2,e_4\}$	0.88	$\{e_2,e_4,e_5\}$	0.72	$\{e_1,e_2,e_3,e_4,e_5\}$	1
$\{e_2,e_4\}$	0.70	$\{e_1,e_2,e_5\}$	0.83	$\{e_3,e_4,e_5\}$	0.82		
$\{e_2,e_5\}$	0.56	$\{e_1,e_3,e_4\}$	0.91	$\{e_1,e_2,e_3,e_4\}$	0.98		

Table 2. The fuzzy measure (weight) of each DM subset

Then new evaluations for DM e_5 are obtained as follows:

$$X^5(2)=\begin{bmatrix} & c_1 & c_2 & c_3 & c_4 & c_5 \\ A_1 & \{H_0,0.13,0.3\} & \{H_{2.7},0.73,0.2\} & \{H_2,0.60,0.2\} & \{H_{2.7},0.67,0.3\} & \{H_{2.7},0.73,0.3\} \\ A_2 & \{H_1,0.33,0.2\} & \{H_{1.7},0.47,0.5\} & \{H_{1.7},0.43,0.4\} & \{H_{1.7},0.47,0.3\} & \{H_{1.7},0.47,0.5\} \\ A_3 & \{H_{1.3},0.43,0.3\} & \{H_{0.7},0.23,0.2\} & \{H_{0.7},0.33,0.1\} & \{H_{0.7},0.30,0.2\} & \{H_{0.3},0.27,0.3\} \\ A_4 & \{H_{2.3},0.63,0.7\} & \{H_{0.7},0.30,0.3\} & \{H_{0.7},0.30,0.1\} & \{H_{0.7},0.30,0.3\} & \{H_0,0.17,0.3\} \end{bmatrix}$$

Based on the new evaluations of DM e_5, the new group decision matrix is:

$$\overline{X}^c=\begin{bmatrix} & c_1 & c_2 & c_3 & c_4 & c_5 \\ A_1 & \{H_0,0.19,0.3\} & \{H_{2.5},0.66,0.2\} & \{H_{2.3},0.63,0.3\} & \{H_{2.1},0.55,0.3\} & \{H_{2.6},0.67,0.3\} \\ A_2 & \{H_{0.6},0.28,0.2\} & \{H_{1.2},0.39,0.5\} & \{H_{1.5},0.42,0.4\} & \{H_{1.6},0.46,0.4\} & \{H_{1.5},0.49,0.5\} \\ A_3 & \{H_{1.9},0.53,0.3\} & \{H_{0.2},0.17,0.4\} & \{H_{0.8},0.32,0.5\} & \{H_{0.6},0.26,0.3\} & \{H_{0.9},0.37,0.6\} \\ A_4 & \{H_{2.5},0.66,0.7\} & \{H_{0.2},0.22,0.7\} & \{H_{0.5},0.25,0.2\} & \{H_{0.5},0.25,0.3\} & \{H_{0.4},0.23,0.4\} \end{bmatrix}$$

The consensus degree of five DMs are computed again as follows: $CD^1=0.8099$, $CD^2=0.7612$, $CD^3=0.7983$, $CD^4=0.7850$, and $CD^5=0.8248$. As the consensus degrees of all five DMs' evaluations have reached the threshold, the consensus reaching process is completed.

Step 7. According to the expert opinions, the fuzzy measures of five criteria are: $\varphi(c_1)=0.14$, $\varphi(c_2)=0.18$, $\varphi(c_3)=0.56$, $\varphi(c_4)=0.22$, and $\varphi(c_5)=0.1$. Then, $\theta_2=-0.43$ is derived by Eq. (2), and the weight vector of five criteria is calculated as follows: $\omega_j=(0.08,0.12,0.49,0.21,0.10)$.

Through the matrix $\overline{X}^c$, based on Eq. (23), the positive (negative) ideal scheme is determined as follows:

$$z^+=\{\{H_{2.5},0.66,0.7\},\{H_{2.5},0.66,0.2\},\{H_{2.3},0.63,0.3\},\{H_{2.1},0.55,0.3\},\{H_{2.6},0.67,0.3\}\},$$

$$z^-=\{\{H_0,0.19,0.3\},\{H_{0.2},0.17,0.7\},\{H_{0.5},0.25,0.2\},\{H_{0.5},0.25,0.3\},\{H_{0.4},0.23,0.4\}\}.$$

According to Eq. (25), the scores of the four grey linguistic numbers are obtained:

$\vartheta_1^+=2.12$; $\vartheta_2^+=1.46$; $\vartheta_3^+=0.77$; $\vartheta_4^+=0.40$; $\vartheta_1^-=-4.57$; $\vartheta_2^-=-13.35$; $\vartheta_3^-=-14.08$; $\vartheta_4^-=-11.58$.

Consequently, it is concluded that $\vartheta_1^->\vartheta_4^->\vartheta_2^->\vartheta_3^-$, so $A_1 \succ A_4 \succ A_2 \succ A_3$. Therefore, the ring cut method A_1 will be more applicable to the section ZK4476 + 964–ZK4477 + 331, which is consistent with the results of previous studies (Xie and Ding, 2019). According to "Mila Mountain Tunnel Engineering Geological Investigation Report" (S13-2-8-1), IV and V grade tuffs are dominated in surrounding rock compositions in the tunnel of Mila Mountain, and the surrounding rock of section ZK4476 + 964–ZK4477 + 331 is V grade tuff. "Construction Organization Design of Highway Reconstruction Project of National Highway 318 Linzhi-Lhasa Section" clearly stipulates that the V grade surrounding rock shall be constructed by the ring cut method. Therefore, it can be proved that the decision result is consistent with the final actual construction choice.

5.1.2 Numerical simulation. In the following subsection, we study the influence of parameters under different values on decision-making process. All the numerical simulation is executed in MATLAB, and the code is written in MATLAB R2019a and all experiments carried out on a Lenovo DESKTOP-AHV6VJ4 desktop computer, running Windows 10 64-bit, with a 3.50 GHz AMD Athlon (tm) X4 855 Quad Core Processor.

The influence of social triangular structure on group consensus is analysed by numerical simulation. The values of parameter α are assigned different DM weights to the Choquet integral operator, which affects the aggregation of group preferences. The consensus levels of five DMs are calculated when $\alpha = 1/6, 2/6, \ldots, 1$, respectively. Figure 5 shows the consensus level of each DM under different parameters.

The experimental results show that: the consensus levels of DMs are acceptable except DM e_5. With the increase of parameter $\alpha \in [0, 1]$, the consensus level of DM e_5 is gradually improved, and the gap between the consensus levels of all DMs is gradually narrowed. This indicates that social triangular structure has significant influence on the consensus reaching process.

For the loss aversion index ξ, we use Matlab to reflect the changes of negative dominance degree ϑ_i^- for each alternative under different values of ξ from 1 to 5. The main results are shown in Figure 6, and it is obvious that the ranking results obtained by different values of ξ are completely same in our case. This indicates the ranking results are not sensitive to the loss aversion index ξ. That is, the proposed method is robust enough to the variation of under different values of ξ, whilst also validating the effectiveness of the ranking method.

5.1.3 Comparison with classical CRP. To demonstrate the effectiveness of the method in this paper, the classical CRP can be used to compare with our proposal and the advantages of the proposed CRP are discussed. To illustrate the importance of considering the trust relationships amongst DMs, the proposed CRP was compared with classical feedback mechanism that failed to consider the trust relationship in social network. The adjustment coefficient is determined by the moderator in the original consensus model and the new

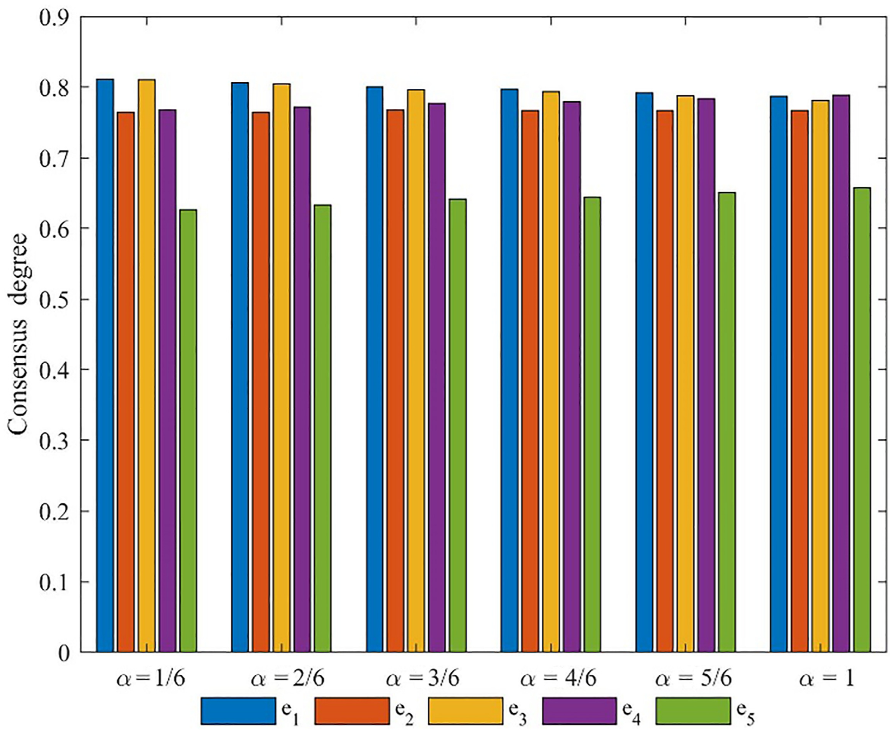

Figure 5.
The consensus degree of each DM for different parameters

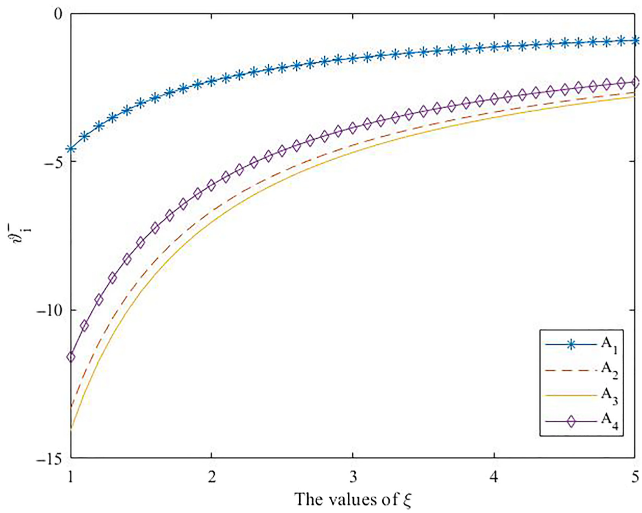

Figure 6.
The negative dominance degree in different values of ξ

decision matrix is obtained by the individual and group weighting method, which is given in Eq. (27).

$$\overline{X}^k(t+1) = (1-\delta)X^k(t) + \delta X^c(t) \tag{27}$$

where $X^k(t)$ and $X^c(t)$ represent the individual decision matrix and group decision matrix, respectively, and $\delta \in (0, 1)$ is the adjustment coefficient determined by the moderator.

In Figure 7, the consensus degree of e_3, e_4 and e_5 increase with an increase in δ whilst the consensus degree of e_1 and e_2 decrease with an increase in δ. In this case, e_5 is advised to modify his/her evaluations. If the value of δ is larger, e_5 needs to drastically change his/her previous evaluations, which will lead to increased cost. The value of the consensus degree of e_1, e_2 decrease when e_5 increases, which means that the CRP may never stop. In addition, when setting the adjustment coefficient, the classical consensus CRP does not consider whether the inconsistent DMs accepts the recommended advices or not. That is, the inconsistent DMs are forced to accept the recommended advices. However, the advantage of group decision-making is to concentrate the wisdom of experts from different fields to deal with increasingly complex decision-making problems. Therefore, it is unreasonable to make the final decision-making without considering the subjectivity of DMs.

To further measure the consensus efficiency, the number of consensus rounds and consensus costs under fixed adjustment coefficients are calculated, as shown in Figures 8 and 9.

The consensus cost can be calculated by:

$$F = \delta\left|X^k(t) - X^c(t)\right| \tag{28}$$

According to Eq. (28), we firstly can calculate the consensus cost of the proposed CRP as 1.1996.

It can be seen from Figures 8 and 9 that whatever the value of δ is, our results are the best compared to the classical feedback mechanism. Specifically, in Figure 8, if the adjustment coefficient δ is set at 0.1, the classical CRP needs to conduct three-round modification. A two-round modification is required when δ takes 0.2~0.3. The multi-round modification will lead

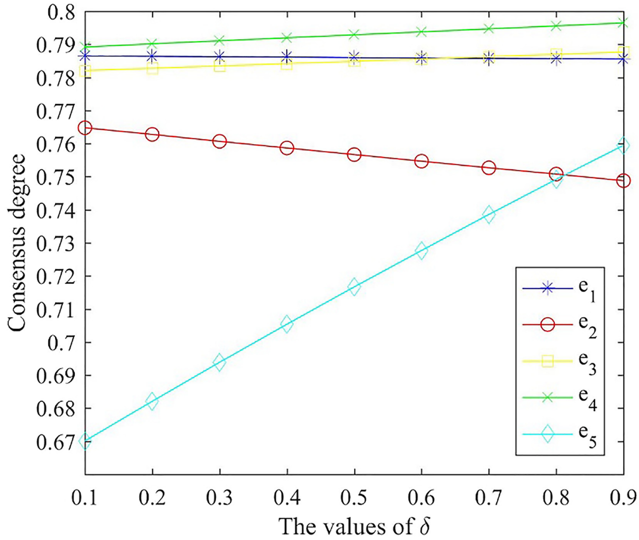

Figure 7.
The consensus degree with different adjustment coefficients

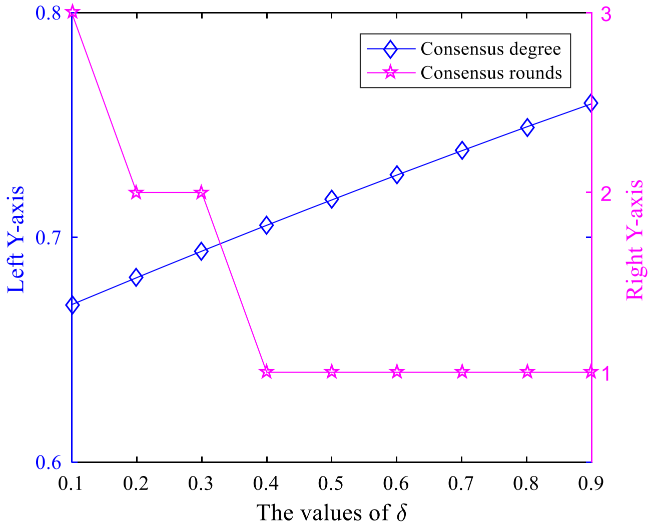

Figure 8.
The consensus rounds and consensus degree of e_5 under different δ

to increased time and costs. Although only one round modification is required when δ is set at 0.4~0.9, consensus costs increase with an increase in δ. It can be seen from Figure 9, when δ takes 0.9, the consensus cost has reached 5.9686, far exceeding the consensus cost in our method. With proposed algorithm, only one round modification and a consensus cost of 1.1996 are needed to make the consensus degree of DM e_5 reach 0.8248. In other words, the novel CRP based on PSO can simplify the consensus process and improve the consensus efficiency. Therefore, from the point of time and cost, the method in this paper is better than the classical methods of discretionary setting the adjustment coefficient.

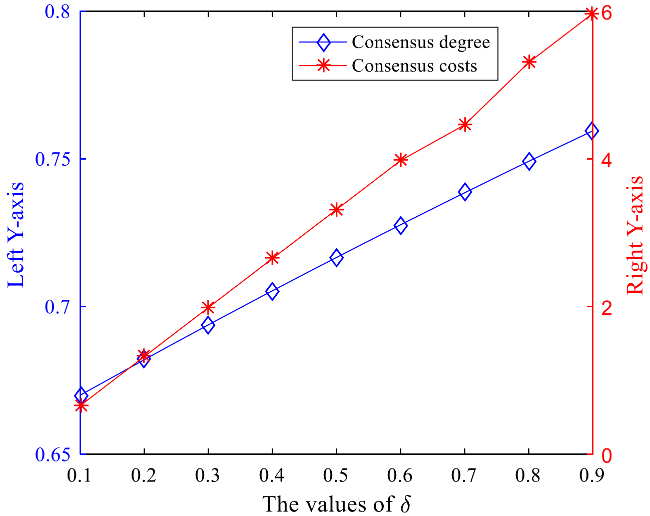

Figure 9.
The consensus costs and consensus degree of e_5 under different δ

5.2 Evaluation of a hotel for the centralised isolation

5.2.1 Case background. In recent years, the frequent occurrence of various types of emergencies such as COVID-19 epidemic has caused major casualties and property loss. As of June 28, 2022, more than 542.18 million people worldwide had been diagnosed with COVID-19. For China, the top priority is reducing the impact of the international epidemic on the domestic epidemic. The effective measure that has been enacted for entry personnel is 14 days of home-based medical observation or centralised isolation with medical observation. Thus, the selection of hotels is an important decision-making problem for epidemic prevention. The scenario of choosing a hotel in Zhengzhou for centralised isolation is simulated. Assume that there are four alternatives, namely A_1: Tongxin Zhengzhou Hotel, A_2: Novotel Hotel (Zhengzhou Airport), A_3: Holiday Inn Express (Zhengzhou Airport), A_4: International Manor Zhengzhou Xinzheng Hotel. Five experts are invited to evaluate four hotels who come from different related fields such as medical treatment, disease prevention and control, comprehensive analysis of the epidemic, and emergency management, etc. They have different professional judgements and trust relationships with others. In this case, we assume that social relationships amongst experts are the same as the case 1, i.e., the weight of each expert is $\lambda_1 = 0.07$, $\lambda_2 = 0.11$, $\lambda_3 = 0.28$, $\lambda_4 = 0.37$, $\lambda_5 = 0.17$. Four criteria are determined for this decision-making problem: cost (c_1), hotel facilities (c_2), hotel environment (c_3), and food (c_4). The weights of the criteria is provided as $\omega_j = (0.35, 0.14, 0.27, 0.24)$.

Step 1. The five decision matrices with IGLNs are shown below:

$$X^1 = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 \\ A_1 & \{H_3, 0.7, 0.1\} & \{H_2, 0.5, 0.2\} & \{H_2, 0.6, 0.8\} & \{H_3, 0.8, 0.7\} \\ A_2 & \{H_3, 0.8, 0.6\} & \{H_3, 0.7, 0.1\} & \{H_2, 0.6, 0.1\} & \{H_2, 0.6, 0.2\} \\ A_3 & \{H_1, 0.4, 0.3\} & \{H_3, 0.8, 0.4\} & \{H_2, 0.5, 0.6\} & \{H_0, 0.2, 0.6\} \\ A_4 & \{H_4, 0.9, 0.7\} & \{H_1, 0.3, 0.2\} & \{H_1, 0.4, 0.1\} & \{H_3, 0.8, 0.3\} \end{bmatrix}$$

$$X^2 = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 \\ A_1 & \{H_3, 0.7, 0.1\} & \{H_1, 0.4, 0.1\} & \{H_2, 0.5, 0.3\} & \{H_3, 0.8, 0.2\} \\ A_2 & \{H_3, 0.7, 0.1\} & \{H_3, 0.8, 0.2\} & \{H_3, 0.8, 0.5\} & \{H_2, 0.5, 0.5\} \\ A_3 & \{H_1, 0.3, 0.5\} & \{H_3, 0.7, 0.5\} & \{H_2, 0.6, 0.3\} & \{H_0, 0.1, 0.6\} \\ A_4 & \{H_3, 0.8, 0.2\} & \{H_0, 0.2, 0.6\} & \{H_2, 0.5, 0.2\} & \{H_3, 0.7, 0.2\} \end{bmatrix}$$

$$X^3 = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 \\ A_1 & \{H_3, 0.7, 0.4\} & \{H_2, 0.5, 0.1\} & \{H_3, 0.7, 0.2\} & \{H_3, 0.7, 0.4\} \\ A_2 & \{H_3, 0.7, 0.3\} & \{H_3, 0.8, 0.3\} & \{H_3, 0.7, 0.4\} & \{H_2, 0.6, 0.6\} \\ A_3 & \{H_1, 0.4, 0.4\} & \{H_4, 0.9, 0.3\} & \{H_2, 0.5, 0.3\} & \{H_1, 0.4, 0.4\} \\ A_4 & \{H_4, 0.9, 0.2\} & \{H_0, 0.1, 0.5\} & \{H_1, 0.4, 0.2\} & \{H_3, 0.8, 0.1\} \end{bmatrix}$$

$$X^4 = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 \\ A_1 & \{H_3, 0.8, 0.1\} & \{H_2, 0.6, 0.3\} & \{H_2, 0.5, 0.6\} & \{H_3, 0.8, 0.3\} \\ A_2 & \{H_3, 0.8, 0.4\} & \{H_3, 0.8, 0.2\} & \{H_2, 0.6, 0.2\} & \{H_2, 0.5, 0.4\} \\ A_3 & \{H_1, 0.4, 0.4\} & \{H_3, 0.8, 0.2\} & \{H_2, 0.6, 0.5\} & \{H_0, 0.2, 0.3\} \\ A_4 & \{H_4, 0.9, 0.2\} & \{H_1, 0.3, 0.4\} & \{H_1, 0.4, 0.3\} & \{H_3, 0.8, 0.3\} \end{bmatrix}$$

$$X^5 = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 \\ A_1 & \{H_2, 0.6, 0.3\} & \{H_3, 0.7, 0.6\} & \{H_2, 0.6, 0.2\} & \{H_3, 0.7, 0.2\} \\ A_2 & \{H_2, 0.6, 0.2\} & \{H_3, 0.7, 0.4\} & \{H_3, 0.8, 0.3\} & \{H_2, 0.5, 0.1\} \\ A_3 & \{H_1, 0.3, 0.1\} & \{H_3, 0.7, 0.2\} & \{H_2, 0.6, 0.6\} & \{H_0, 0.2, 0.2\} \\ A_4 & \{H_3, 0.7, 0.6\} & \{H_1, 0.4, 0.1\} & \{H_2, 0.5, 0.2\} & \{H_3, 0.7, 0.3\} \end{bmatrix}$$

Step 2. The social relationships amongst experts are the same as the case 1, i.e., the weight of each expert is $\lambda_1 = 0.07$, $\lambda_2 = 0.11$, $\lambda_3 = 0.28$, $\lambda_4 = 0.37$, $\lambda_5 = 0.17$.

Step 3. The group decision matrix is calculated by Eq. (12).

$$X^c = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 \\ A_1 & \{H_{2.8}, 0.72, 0.4\} & \{H_{2.1}, 0.56, 0.6\} & \{H_{2.3}, 0.58, 0.8\} & \{H_3, 0.76, 0.7\} \\ A_2 & \{H_{2.8}, 0.73, 0.6\} & \{H_3, 0.78, 0.4\} & \{H_{2.6}, 0.68, 0.5\} & \{H_2, 0.54, 0.6\} \\ A_3 & \{H_1, 0.37, 0.5\} & \{H_{3.3}, 0.8, 0.5\} & \{H_2, 0.57, 0.6\} & \{H_{0.3}, 0.25, 0.6\} \\ A_4 & \{H_{3.7}, 0.86, 0.7\} & \{H_{0.6}, 0.25, 0.6\} & \{H_{1.3}, 0.43, 0.3\} & \{H_3, 0.77, 0.3\} \end{bmatrix}$$

Step 4. The consensus degree of five DMs are computed based on Eqs. (14)-(16), and the results are $CD^1 = 0.8050$, $CD^2 = 0.6901$, $CD^3 = 0.7175$, $CD^4 = 0.8053$, and $CD^5 = 0.5897$. Due to $CD^5 < 0.6700$, therefore, DM e_5 needs to modify her/his evaluations to reach the threshold.

Step 5. The adjustment coefficients for e_5 are determined using Eq. (20) as follows: $\delta_{51} = 0.33$ $\delta_{52} = 0$, $\delta_{53} = 0$, $\delta_{54} = 0$, $\delta_{55} = 0.67$.

Step 6. According to Eqs. (20)-(21), Then new evaluations for DM e_5 are obtained as follows:

$$X^5(2) = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 \\ A_1 & \{H_{2.3}, 0.63, 0.3\} & \{H_{2.7}, 0.63, 0.6\} & \{H_2, 0.60, 0.2\} & \{H_3, 0.73, 0.2\} \\ A_2 & \{H_{2.3}, 0.67, 0.2\} & \{H_3, 0.70, 0.4\} & \{H_{2.7}, 0.73, 0.3\} & \{H_2, 0.53, 0.1\} \\ A_3 & \{H_1, 0.33, 0.1\} & \{H_3, 0.73, 0.2\} & \{H_2, 0.57, 0.6\} & \{H_0, 0.20, 0.2\} \\ A_4 & \{H_{3.3}, 0.77, 0.6\} & \{H_1, 0.37, 0.1\} & \{H_{1.7}, 0.47, 0.2\} & \{H_3, 0.73, 0.3\} \end{bmatrix}$$

Based on the new evaluations of DM e_5, the new group decision matrix is:

$$\overline{X}^c = \begin{bmatrix} & c_1 & c_2 & c_3 & c_4 \\ A_1 & \{H_{2.9}, 0.73, 0.4\} & \{H_2, 0.55, 0.6\} & \{H_{2.3}, 0.58, 0.8\} & \{H_3, 0.76, 0.7\} \\ A_2 & \{H_{2.9}, 0.74, 0.6\} & \{H_3, 0.78, 0.4\} & \{H_{2.5}, 0.67, 0.5\} & \{H_2, 0.54, 0.6\} \\ A_3 & \{H_1, 0.38, 0.5\} & \{H_{3.3}, 0.81, 0.5\} & \{H_2, 0.56, 0.6\} & \{H_{0.3}, 0.25, 0.6\} \\ A_4 & \{H_{3.8}, 0.87, 0.7\} & \{H_{0.6}, 0.24, 0.6\} & \{H_{1.2}, 0.42, 0.3\} & \{H_3, 0.78, 0.3\} \end{bmatrix}$$

The consensus degree of five DMs are computed again as follows: $CD^1 = 0.8280$, $CD^2 = 0.6827$, $CD^3 = 0.7291$, $CD^4 = 0.8165$, and $CD^5 = 0.7258$. The consensus reaching process is completed.

Step 7. Through the matrix $\overline{X}^c$, based on Eq. (23), the positive (negative) ideal scheme is determined as follows:

$$z^+ = \{\{H_{3.8}, 0.87, 0.7\}, \{H_{3.3}, 0.81, 0.5\}, \{H_{2.5}, 0.67, 0.5\}, \{H_3, 0.78, 0.3\}\},$$

$$z^- = \{\{H_1, 0.38, 0.5\}, \{H_{0.6}, 0.24, 0.6\}, \{H_{1.2}, 0.42, 0.3\}, \{H_{0.3}, 0.25, 0.6\}\}.$$

According to Eq. (25), the scores of the four grey linguistic numbers are obtained:

$\vartheta_1^+ = 2.34$; $\vartheta_2^+ = 2.39$; $\vartheta_3^+ = 0.99$; $\vartheta_4^+ = 1.74$; $\vartheta_1^- = -6.77$; $\vartheta_2^- = -5.59$; $\vartheta_3^- = -7.39$; $\vartheta_4^- = -6.30$.

Consequently, it is concluded that $\vartheta_2^- > \vartheta_4^- > \vartheta_1^- > \vartheta_3^-$, so $A_2 \succ A_4 \succ A_1 \succ A_3$. Thus, the optimal hotel is A_2 : the Novotel Hotel (Zhengzhou Airport).

5.2.2 Numerical simulation. In this subsection, we investigate the influence of different ξ from 1 to 5 on ranking results. As seen from Figure 10, we find that the best alternative is always A_2. When ξ from 1 to 5, the ranking order remains unchanged, i.e. $A_2 \succ A_4 \succ A_1 \succ A_3$, which explains the stability of this method to some extent.

5.2.3 Comparison with classical CRP. In the following, δ takes nine numbers in steps of 0.1 from 0 to 1. The new evaluations of e_5 are calculated separately using different adjustment coefficients. The consensus degree with the new evaluations are shown in Figure 11. In this section, the number of consensus rounds and consensus costs are also applied to show the effectiveness of our methods. In addition, we also give the results of all the threshold values in the group with respect to the threshold 0.67.

In Figure 11, the consensus degree for e_5 does not reach the threshold when $\delta = 0.1$ and $\delta = 0.2$, so a new evaluation round is needed. The consensus degree of e_1, e_3, e_4 and e_5 increase with an increase in δ whilst the consensus degree of e_2 decrease with an increase in δ. In this case, when e_5 is asked to change his/her evaluations, if the value for δ is larger, the consensus degree of e_5 is larger. Therefore, the larger the adjustment coefficient, the easier it is to achieve the threshold. However, a multi-round evaluation and an inappropriate adjustment coefficient could lead to increased time and costs.

To further measure the consensus efficiency, consensus costs under fixed adjustment coefficients are calculated, as shown in Figure 12. For comparison, the consensus adjustment costs of the proposed method could be obtained as 1.94.

It can be seen from Figure 12, consensus costs increase with an increase in δ. When $\delta > 0.2$, our consensus costs are the lowest compared to the classical CRP. when $\delta = 0.1$ and $\delta = 0.2$, although the consensus costs of classical CRP are lower than 1.94, the consensus degree of e_5 does not reach the threshold. It means that a new round modification is needed. The final number of consensus rounds and consensus costs under fixed adjustment coefficients are calculated, as shown in Figures 13 and 14. From Figure 13, when $\delta = 0.1$ and

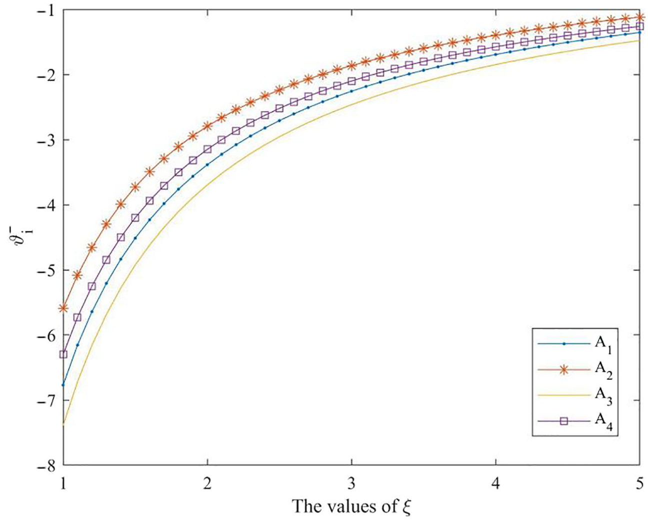

Figure 10. The negative dominance degree in different values of ξ

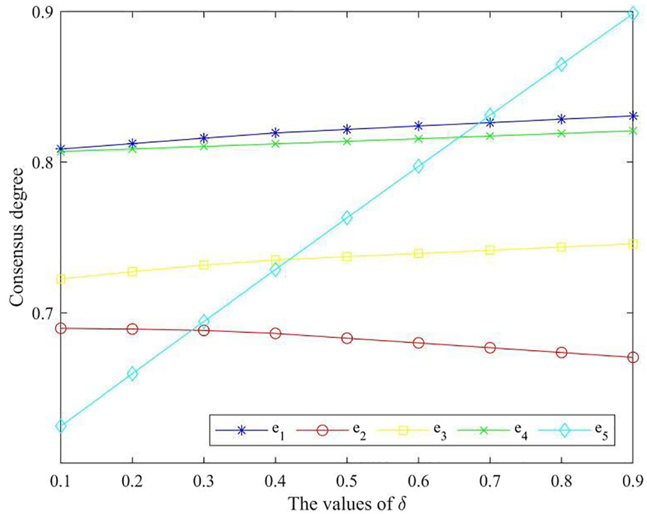

Figure 11. The consensus degree with different adjustment coefficients

$\delta = 0.2$, although consensus degree of e_5 reached the threshold, the number of consensus rounds are more than the others, which indicated that more costs are needed from Figure 14, when $\delta = 0.1$, the final consensus costs are 1.9632; when $\delta = 0.2$, the final consensus costs are 2.0362. Therefore, whatever the value of δ is, our results are the best compared to the classical CRP.

According to the analysis of above two practical applications, the advantages of the proposed method can be summarised as follows:

(1) A novel DM weight calculation method based on the social triangle structure is proposed, which considers the triangle structure of DMs' social network and scale of adjacent DMs.

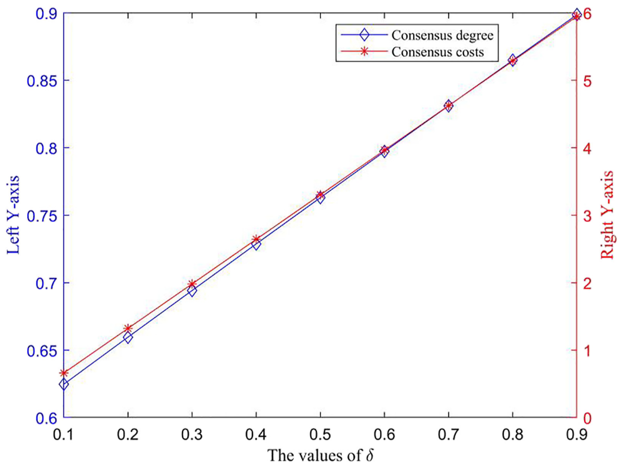

Figure 12.
The initial consensus costs and consensus degree of e_5 under different δ

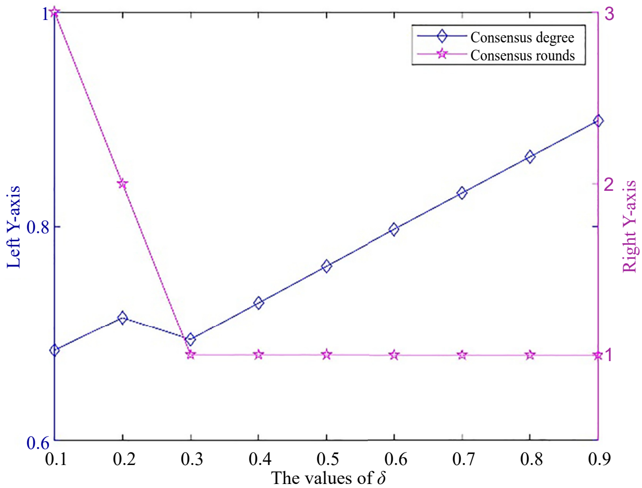

Figure 13.
The consensus rounds and consensus degree of e_5 under different δ

(2) A polygon area-based calculation method is developed to measure the consensus level, which can reflect the proximity of curve on the distance and the similarity of the geometry shape.

(3) The PSO algorithm is used to optimise the decision-making process, which can not only simplify the decision-making process, but also improve the consensus level of decision-maker. Moreover, compared with the classical CRP model, it can be found that the proposed method reduces the time and cost of decision-making.

6. Conclusions

This paper mainly explores the CRP based on IGLNs in MCGDM. The DMs' triangle structure and their adjacent DMs in social network are analysed by the theory of structural hole and

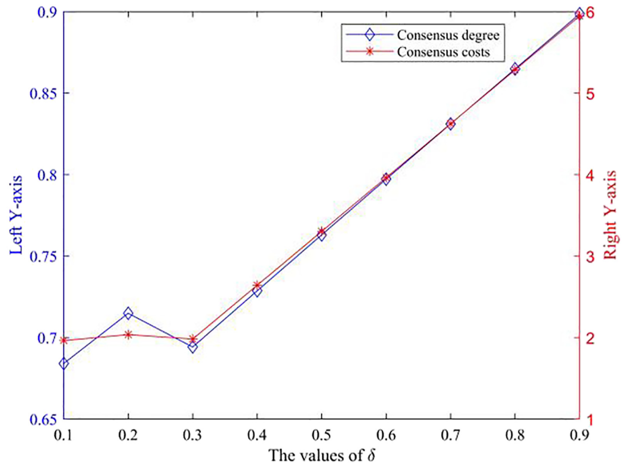

Figure 14.
The final consensus costs and consensus degree of e_5 under different δ

three degrees of influence rule, and a novel weight determination method for DMs is proposed which takes into account the triangular structure between DMs and the degree centrality of adjacent DMs. All inconsistent DMs are identified by three-level polygon area. To reflect the DMs' subjectivity and simplify consensus process in MCGDM, the PSO algorithm with IGLNs is developed to derive the adjustment coefficients and strategy for the inconsistent DMs. Numerical simulation and comparison analysis are given to further illustrate the superiority and feasibility of this method.

The main conclusions of this paper are as follows.

(1) The DMs' weights in social network are not only influenced by the structure between a DM and his/her adjacent DMs, but also by the scale of adjacent DMs.

(2) The consensus measure is not only related to the distance of corresponding criteria between individual decision matrix and group matrix, but also related to each adjacent criteria. Therefore, interactions amongst criteria are taken into account when calculating the consensus degree of DMs.

(3) The consensus mechanism is optimised by the PSO algorithm considering trust relationships, which reserves the subjectivity of DMs and reduces the time and cost of decision-making.

Currently, a large number of consensus algorithms are included in the blockchain system such as PoW, PoS, DPoS, etc. It will be very challenging and interesting to consider the effect of blockchain technology on consensus reaching process in future studies.

References

Burt, R.S. (1992), *Structural Holes: The Social Structure of Competition*, Harvard University Press, Cambridge.

Cheng, D., Zhou, Z.L., Cheng, F.X, Zhou, Y.F. and Xie, Y.J. (2018), "Modeling the minimum cost consensus problem in an asymmetric costs context", *European Journal of Operational Research*, Vol. 270 No. 3, pp. 1122-1137.

Cheng, D., Cheng, F.X., Zhou, Z.L. and Wu, Y. (2020), "Reaching a minimum adjustment consensus in social network group decision-making", *Information Fusion*, Vol. 59, pp. 30-43.

Christakis, N.A. and Fowler, J.H. (2009), *Connected: the Surprising Power of Our Social Networks and How They Shape Our Lives*, Little Brown, New York.

Chu, J.F., Liu, X.W. and Wang, Y.M. (2016), "Social network analysis based approach to group decision making problem with fuzzy preference relations", *Journal of Intelligent and Fuzzy Systems*, Vol. 31 No. 3, pp. 1271-1285.

Eberhart, R. and Kennedy, J. (1995), "Particle swarm optimization", *Proceedings of the IEEE International Conference on Neural Networks*, Citeseer, Vol. 4, pp. 1942-1948.

Garg, H. and Rani, D. (2021), "An efficient intuitionistic fuzzy MULTIMOORA approach based on novel aggregation operators for the assessment of solid waste management techniques", *Applied Intelligence*, Vol. 52 No. 4, pp. 4330-4363.

Gou, X. and Xu, Z. (2021), *Double Hierarchy Linguistic Term Set and its Extensions: Theory and Methods*, 1st ed., Springer Nature Switzerland AG.

Han, E.D., Guo, P. and Zhao, J. (2016a), "Method for multi-attribute group decision making based on interval grey uncertain linguistic information", *Journal of Frontiers of Computer Science and Technology*, Vol. 10 No. 01, pp. 93-102.

Han, Z.M., Chen, Y. and Li, M.Q. (2016b), "An efficient node influence metric based on triangle in complex networks", *Acta Physica Sinica*, Vol. 65 No. 16, pp. 289-300.

Herrera-Viedma, E., Cabrerizo, F.J., Kacprzyk, J. and Witold, P. (2014), "A review of soft consensus models in a fuzzy environment", *Information Fusion*, Vol. 17, pp. 4-13.

Hou, X.Y. and Tong, J. (2021), "Consensus-building algorithm based on fuzzy group decision making for emergency epidemic assessment", *Statistics and Decision*, Vol. 37 No. 12, pp. 185-188.

Jiang, S.Q., Liu, S.F., Liu, Z.X. and Fang, Z.G. (2015), "Grey incidence decision making model based on area", *Control and Decision*, Vol. 30 No. 04, pp. 685-690.

Jiang, S.Q., Liu, S.F. and Liu, Z.X. (2020), "Grey multi-attribute decision-making method based on intuitionistic grey number set", *Statistics and Information Forum*, Vol. 35 No. 03, pp. 3-8.

Jung, S., Lee, S., Kwon, O. and Kim, B. (2020), "Grid-based traffic vulnerability analysis by using betweenness centrality", *Journal of the Korean Physical Society*, Vol. 77 No. 7, pp. 538-544.

Kamis, N.H., Chiclana, F. and Levesley, J. (2018), "Preference similarity network structural equivalence clustering based consensus group decision making model", *Applied Soft Computing*, Vol. 67, pp. 706-720.

Kundu, S. and Pal, S.K. (2015), "FGSN: fuzzy granular social networks-model and applications", *Information Science*, Vol. 314, pp. 100-117.

Lee, J.S., Lee, Y.S., Oh, S.M. and Kahng, B. (2021), "Betweenness centrality of teams in social networks", *Chaos (Woodbury, N.Y.)*, Vol. 31 No. 6, p. 061108.

Li, C.B. and Yuan, J.H. (2017), "A new multi-attribute decision-making method with three-parameter interval grey linguistic variable", *International Journal of Fuzzy Systems*, Vol. 19 No. 2, pp. 292-300.

Li, Y.M., Wu, C.T. and Lai, C.Y. (2013), "A social recommender mechanism for e-commerce: combining similarity, trust, and relationship", *Decision Support Systems*, Vol. 55, pp. 740-752.

Li, Q.S., Diao, Y.Z., Gong, Z.W. and Hu, A. (2018), "Grey language hesitant fuzzy group decision making method based on kernel and grey scale", *International Journal of Environmental Research and Public Health*, Vol. 15 No. 3, p. 436.

Liang, Q., Liao, X.W. and Liu, J.P. (2017), "A social ties-based approach for group decision-making problems with incomplete additive preference relations", *Knowledge-Based Systems*, Vol. 119, pp. 68-86.

Liu, S.F. (2017), *Grey System Theory and its Applications*, 8th ed., Science Press, Beijing.

Liu, Y.Y. and Yang, Y.L. (2022), "An extended VIKOR method based on particle swarm optimization and novel operations of probabilistic linguistic term sets for multicriteria group decision-making problem", *International Journal of Intelligent Systems*, Vol. 37 No. 8, pp. 5381-5424.

Liu, S.F., Li, Q.S. and Zhao, N. (2016), "Grey correlation decision-making method of grey hesitant fuzzy sets based on kernel and grey degree", *Journal of Nanjing University of Aeronautics and Astronautics*, Vol. 48 No. 5, pp. 683-688.

Liu, Y.J., Liang, C.Y., Chiclana, F. and Wu, J. (2017), "A trust induced recommendation mechanism for reaching consensus in group decision making", *Knowledge-Based Systems*, Vol. 119, pp. 221-231.

Luis, G.P., Francisco, M. and Francisco, C. (2014), "Social network decision making with linguistic trustworthiness-based induced OWA operators", *International Journal of Intelligent Systems*, Vol. 29 No. 12, pp. 1117-1137.

May, R.M. (1976), "Simple mathematical models with very complicated dynamics", *Nature*, Vol. 261 No. 5560, pp. 459-467.

Murofushi, T. and Sugeno, M. (1989), "An interpretation of fuzzy measures and the Choquet integral as an integral with respect to a fuzzy measure", *Fuzzy Sets and Systems*, Vol. 29 No. 2, pp. 201-227.

Samadi, M., Nikolaev, A. and Nagi, R. (2016), "A subjective evidence model for influence maximization in social networks", *Omega*, Vol. 59, pp. 263-278.

Wang, F., Du, D.Y. and Zhao, J. (2019), "A novel SIR center dot Choquet method for multiple attributes group decision-making with interval grey linguistic", *International Journal of Fuzzy Systems*, Vol. 21 No. 6, pp. 1771-1785.

Wu, J. (2009), *Study on Grey Language Multi-Criteria Decision-Making Method and its Application*, Central South University, Changsha.

Wu, J., Dai, L.F., Chiclana, F., Fujita, H. and Herrera-Viedma, E. (2018), "A minimum adjustment cost feedback mechanism based consensus model for group decision making under social network with distributed linguistic trust", *Information Fusion*, Vol. 41, pp. 232-242.

Wu, Z.B., Huang, S. and Xu, J.P. (2019), "Multi-stage optimization models for individual consistency and group consensus with preference relations", *European Journal of Operational Research*, Vol. 275 No. 1, pp. 182-194.

Wu, T., Liu, X.W., Gong, Z.W., Zhang, H.H. and Herrera, F. (2020), "The minimum cost consensus model considering the implicit trust of opinions similarities in social network group decision-making", *International Journal of Intelligent Systems*, Vol. 35 No. 3, pp. 470-493.

Xie, Q.M. and Ding, X.Y. (2019), "Optimization of tunnel construction method based on grey fuzzy multi-attribute group decision-making", *Journal of Yangtze River Scientific Research Institute*, Vol. 36 No. 05, pp. 1-6.

Yan, B., Wang, Y.J. and Jin, P. (2022), "An improved method for satellite emergency mission scheduling scheme group decision-making incorporating PSO and MULTIMOORA", *Journal of Intelligent and Fuzzy Systems*, Vol. 42 No. 4, pp. 3837-3853.

Yu, L. and Lai, K.K. (2011), "A distance-based group decision-making methodology for multi-person multi-criteria emergency decision support", *Decision Support Systems*, Vol. 51 No. 2, pp. 307-315.

Zadeh, L.A. (1965), "Fuzzy sets", *Information and Control*, Vol. 8 No. 3, pp. 338-366.

Zhang, Z. and Pedrycz, W. (2018), "Goal programming approaches to managing consistency and consensus for intuitionistic multiplicative preference relations in group decision making", *IEEE Transactions on Fuzzy Systems*, Vol. 26 No. 6, pp. 3261-3275.

Zhang, X.L. and Xu, Z.S. (2015), "Soft computing based on maximizing consensus and fuzzy TOPSIS approach to interval-valued intuitionistic fuzzy group decision making", *Applied Soft Computing Journal*, Vol. 26, pp. 42-56.

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